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Geometry dependent resistor model for predicting effective thermal conductivity of two phase systems

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1. INTRODUCTION

THE KNOWLEDGE of the effective thermal conductivity of heterogeneous materials such as soils, ceramics, fiber reinforced materials and composites are becoming increasingly important in the technological developments and in many applications. Dependence of the effective thermal conductivity (*ETC*) of these materials on porosity, grain size and shape of the particles is also a matter of concern to engineers, architects and physicists. As it is not often possible to conduct experiments to study the effect of the above parameters on the *ETC*, a theoretical expression is needed to predict its value.

Though a large number of models exist in the literature, a general expression which can predict λ_e (*ETC*) of all kinds of two phase systems with the above parameters is still lacking.

The present paper is an effort to find a suitable expression to predict the *ETC* of various kinds of two phase systems. We have taken the electrical analog of various parameters to develop the expression. Equivalent thermal resistors formed out of the phases in form of parallel slabs are considered and the resistor model approach has been applied. The slabs are taken to be inclined to the direction of heat flow. By varying the angle of the slabs, the *ETC* of different two phase materials can be predicted. The angle has been defined in terms of various structural and thermal parameters.

2. THEORY

On the basis of phase averaging of temperature field, the following closure equations can be written for a two phase system. According to Hadley [1],

$$\nabla\langle T\rangle = \phi\langle\nabla T_1\rangle^1 + (1-\phi)\langle\nabla T_2\rangle^2 \tag{1}$$

$$\frac{\lambda_e}{\lambda_1}\nabla\langle T\rangle = \phi\langle\nabla T_1\rangle^1 + \frac{\lambda_2}{\lambda_1}(1-\phi)\langle\nabla T_2\rangle^2 \tag{2}$$

where $\langle\nabla T_1\rangle^1$ and $\langle\nabla T_2\rangle^2$ are average of the gradients in continuous phase and dispersed phase, respectively. ϕ is the porosity (volume fraction of continuous phase). These two equations contain three parameters $\nabla\langle T\rangle$, $\langle\nabla T_1\rangle^1$, and $\langle\nabla T_2\rangle^2$ and hence cannot be solved unless some relation connecting these parameters is assumed.

One possibility is $\langle\nabla T_1\rangle^1 = \langle\nabla T_2\rangle^2$, i.e. average temperature gradients in the two phases are equal. This condition is met in a collection of phase slabs, parallel to the direction of heat flow. This equality when put in equations (1) and (2) gives

$$\lambda_{\parallel} = [\phi\lambda_1 + (1-\phi)\lambda_2]. \tag{3}$$

This is an expression for equivalent thermal conductivity of resistors arranged in parallel.

Similarly the assumption

$$\langle\nabla T_1\rangle^1 = \frac{\lambda_2}{\lambda_1}\langle\nabla T_2\rangle^2$$

when put in equations (1) and (2) gives,

$$\lambda_{\perp} = \left[\frac{\phi}{\lambda_1} + \frac{(1-\phi)}{\lambda_2}\right]^{-1}. \tag{4}$$

It is an expression for equivalent thermal conductivity of resistors arranged perpendicular to the heat flow. The above condition is equivalent to $\lambda_1\langle\nabla T_1\rangle^1 = \lambda_2\langle\nabla T_2\rangle^2$, i.e. the heat flux passing through different phases is the same. It is a situation met with the slabs perpendicular to the direction of heat flow.

Any model for a two phase system, having the *ETC* dependent on ϕ and λ_2/λ_1 can be represented by a general equation,

$$\langle\nabla T_1\rangle^1 = \left[f + \frac{\lambda_2}{\lambda_1}(1-f)\right]\langle\nabla T_2\rangle^2 \tag{5}$$

where f is a parameter lying between 0 and 1.

Here $\lambda_{\parallel}$ and $\lambda_{\perp}$ also represent upper and lower bounds of the effective thermal conductivity for a mixture.

Thus $\lambda_{\parallel} = (\lambda_e)_{\max}$ and $\lambda_{\perp} = (\lambda_e)_{\min}$.

We know that a porous medium is neither composed of slabs parallel to the heat flux nor perpendicular to it, yet the concept of the slabs is capable of predicting the maximum and minimum limits of the *ETC*. Therefore, it is proposed that the slabs of the continuous and dispersed phases, inclined to the heat flux may represent the *ETC* of the system.

NOMENCLATURE

A	cross-sectional area
f	parameter used by Hadley ($0 < f < 1$)
F	resistivity formation factor
p	factor converting three dimensional porosity to its equivalent in two dimensions
s	surface area of a sphere having same volume as that of the particle
S	actual surface area of the particle.

Greek symbols

θ	angle of inclination of slabs with respect to the direction of heat flow
λ	thermal conductivity
ϕ	porosity (volume fraction of continuous phase)
ψ	sphericity of particles.

Subscripts and superscripts

av	average value
e	effective value
max	maximum value
min	minimum value
pl	component parallel to heat flux
pr	component perpendicular to heat flux
1	continuous phase
2	dispersed phase
$\parallel$	mode parallel to heat flux
$\perp$	mode perpendicular to heat flux.

Now, let us assume the continuous and the dispersed phases in form of parallel slabs (equivalent resistors), which make an angle 'θ' with the direction of heat flux. Let us also assume the *ETC* to have a direction along the slabs. As the slabs are neither parallel not perpendicular to the heat flux, we resolve the *ETC* (λ_e) into two components, one parallel to the heat flux (say λ_{pl}) and another perpendicular to it (λ_{pr}). The two components should be such that,

(i) For $\theta = 0$, the component λ_{pl} reduces to $\lambda_{\parallel} = (\lambda_e)_{max}$ and that λ_{pr} reduce to $\lambda_{\perp} = 0$.

(ii) Also for $\theta = \pi/2$, we should have $\lambda_{pl} = \lambda_{\parallel} = 0$ and $\lambda_{pr} = \lambda_{\perp} = (\lambda_e)_{min}$.

These considerations lead to the conditions that the components should be,

$$\lambda_{pl} = (\lambda_e)_{max} \cos\theta$$

and

$$\lambda_{pr} = (\lambda_e)_{min} \sin\theta.$$

The resolution of the *ETC* (λ_e) also gives,

$$\lambda_{pl} = \lambda_e \cos\theta \tag{6}$$

and

$$\lambda_{pr} = \lambda_e \sin\theta \tag{7}$$

hence, the effective thermal conductivity should be,

$$\lambda_e = [\lambda_{pl}^2 + \lambda_{pr}^2]^{1/2}. \tag{8}$$

The equations (6)–(8) above suggest that an increase in angle θ will increase λ_{pr} and decrease λ_{pl} components. The net result will be a decrease in λ_e. On the other hand, decrease in θ will have a reverse effect and λ_e will increase.

Thus by knowing the angle of inclination of the slabs 'θ', the *ETC* of the system can be obtained. To interpolate the *ETC* of various systems between two limits, the angle has to be defined in terms of various structural and thermal parameters.

3. MATHEMATICAL FORMULATION FOR ANGLE θ

In addition to the physical parameters such as porosity, temperature and interstitial pressure, the *ETC* of a two phase system is also found to depend upon various other parameters. It is likely that the angle θ will be a function of these parameters. Such parameters are given in the subsections below.

3.1. *Resistivity formation factor* (*F*)

This factor was first introduced by Archie [2] for accounting the flow of electrical energy in a porous medium. It is defined as the ratio of electrical resistivity of a porous medium fully saturated with an electrolyte to the resistivity of the electrolyte itself. On the same analogy it can also be defined for the flow of thermal energy in a porous medium. Thus,

$$F = \frac{\text{Thermal resistance with inclusion of particles in the medium}}{\text{Thermal resistance of the medium itself}}.$$

It is a measure of the average path traversed by the lines of flux in a porous system. Its value is given by Agrawal and Bhandari [3]

$$F = \frac{\left[1 + \frac{0.3219(1-\phi)}{\psi}\right]^2}{\phi}. \tag{9}$$

The resistivity formation factor is independent of the thermal conductivity of the two phases and it has no dimensions. Its value is always greater than one.

This factor is widely used in the petroleum industry to characterise the pore structure of sedimentary rocks and shales. A detailed account of this factor is given by Wyllie and Gregory [4].

Authors Woodside and Messmer [5], Chaudhary and Bhandari [6] have shown that the thermal conductivity of sand and sand stones does depend upon the formation factor. We feel that the thermal conductivity of a porous medium should depend upon average deviation of the flux lines in the medium. The greater the deviation, the higher should be the thermal resistivity and the lower the thermal conductivity. For no dispersion ($\phi = 1$), the flux lines will remain parallel and we have $F = 1$.

3.2. *Sphericity of the particles* (ψ)

Angularities of the grains in a porous system greatly alter its thermophysical properties. Experiments show that the behaviour of a system packed with non-spherical particles is radically different to that involving spherical particles. A good account of the dependence of physical properties on angularities of particles is given by Haughey and Baveridge [7].

Attempts have been made to describe the deviation of particle shape from a sphere by a factor called 'sphericity'. It is a measure of the roundness of the particles. Wadell [8] has defined it as $\psi = (s/S)$, where s is the surface area of a sphere having same volume as that of the particle and S being its actual surface area. For spherical particles $\psi = 1$ and for particles of any other shape $\psi < 1$. Wadell has also described a method for its experimental determination. The

Table 1. Experimental values of *ETC* and other data for porous granular systems. Values of thermal conductivities are in W m^{-1} K^{-1}

SN	Sample Solid phase/fluid phase	Th. cond. solid (λ_2)	Th. cond. fluid (λ_1)	*ETC* exp (λ_e)	Porosity (ϕ)	Sphericity (ψ)	Ref.
1	Glass sphere/air	1.099	0.024	0.227	0.260	1.00	[14]
2	Glass sphere/air	1.099	0.024	0.041	0.800	1.00	[14]
3	Silica sphere/water	12.414	0.586	2.544	0.431	1.00	[15]
4	Stainless steel/eth. alcohol	20.864	0.337	2.009	0.505	1.00	[15]
5	Glass sphere/iso-octane	1.061	0.144	0.406	0.430	1.00	[15]
6	Lead shots/helium	34.347	0.147	2.140	0.380	0.82	[5]
7	Lead shots/hydrogen	34.347	0.179	2.429	0.380	0.82	[5]
8	Lead shots/water	34.347	0.627	5.404	0.380	0.82	[5]
9	Zircona powder/air	2.001	0.021	0.120	0.530	0.82	[16]
10	Lead/water	33.764	0.586	4.329	0.400	0.82	[17]
11	Zircona powder/air	2.001	0.030	0.230	0.420	0.82	[18]
12	Zircona powder/air	2.001	0.030	0.281	0.360	0.82	[18]
13	Zircona powder/air	2.001	0.030	0.364	0.300	0.82	[18]
14	Glass beads/air	1.201	0.028	0.220	0.350	0.82	[19]
15	Glass beads/benzene	1.201	0.140	0.500	0.350	0.82	[19]
16	Quartz sand/water	5.003	0.620	2.331	0.324	0.82	[19]
17	Glass beads/air	1.091	0.029	0.180	0.400	0.82	[20]
18	Microbeads/air	1.046	0.026	0.193	0.350	0.82	[21]
19	Microbeads/soltrol	1.046	0.133	0.452	0.361	0.82	[21]
20	Wassau sand/*n*-heptane	8.374	0.129	0.722	0.515	0.82	[5]
21	Ottawa sand/helium	8.374	0.147	1.323	0.360	0.82	[5]
22	Wassau sand/helium	8.374	0.147	0.598	0.590	0.82	[5]
23	Miami silt loam/air	2.932	0.023	0.169	0.544	0.82	[22]
24	Miami silt loam/air	2.932	0.023	0.221	0.448	0.82	[22]
25	Nanaura sand stones/water	1.973	0.601	1.450	0.220	0.82	[14]
26	Glass/air	1.130	0.026	0.176	0.400	0.87	[23]
27	Glass/air	1.130	0.026	0.181	0.400	0.82	[23]
28	Glass/air	1.130	0.026	0.190	0.400	0.73	[23]
29	Glass/air	1.130	0.026	0.197	0.400	0.64	[23]
30	Glass/air	1.130	0.026	0.207	0.400	0.59	[23]

Calculation-table. Values of *ETC* are in W m^{-1} K^{-1}

SN	*ETC* exp (λ_e)	Proposed model equation (8) *ETC*	%Err	Brailsford model equation (A1) *ETC*	%Err	Litchnecker model equation (A2) *ETC*	%Err	Hadley's model equation (A3) *ETC*	%Err	Pande's model equation (A4) *ETC*	%Err
1	0.227	0.264	16.3	0.186	18.0	0.406	79.0	0.280	23.3	0.051	77.7
2	0.041	0.045	7.4	0.040	2.6	0.051	23.9	0.053	26.9	0.042	1.4
3	2.544	2.505	1.6	2.312	9.1	3.330	30.9	3.209	26.1	1.410	44.6
4	2.009	1.807	10.0	1.240	38.3	2.598	29.3	1.892	5.9	1.641	18.3
5	0.406	0.401	1.3	0.417	2.7	0.449	10.7	0.494	21.6	0.238	41.4
6	2.140	2.302	7.5	0.843	60.6	4.323	102.0	1.400	34.6	0.879	58.9
7	2.429	2.544	4.7	1.019	58.0	4.657	91.7	1.684	30.7	0.979	59.7
8	5.404	4.901	9.3	3.304	38.9	7.503	38.8	5.097	5.7	1.989	63.2
9	0.120	0.131	9.0	0.073	38.6	0.179	49.1	0.114	5.0	0.114	5.2
10	4.329	4.387	1.3	2.915	32.7	6.672	54.1	4.508	4.1	2.010	53.6
11	0.230	0.224	2.6	0.141	38.7	0.342	48.6	0.220	4.3	0.116	49.6
12	0.281	0.274	2.4	0.171	39.3	0.440	56.5	0.268	4.6	0.097	65.5
13	0.364	0.342	6.0	0.211	42.1	0.566	55.6	0.331	9.0	0.081	77.8
14	0.220	0.215	2.3	0.156	29.2	0.321	45.9	0.235	7.0	0.074	66.5
15	0.500	0.509	1.8	0.506	1.1	0.566	13.1	0.606	21.2	0.218	56.5
16	2.331	2.334	0.1	2.300	1.3	2.544	9.1	2.718	16.6	0.923	60.4
17	0.180	0.178	1.0	0.137	24.0	0.255	41.8	0.204	13.1	0.083	54.1
18	0.193	0.195	0.9	0.145	25.0	0.287	48.7	0.217	12.3	0.067	65.2
19	0.452	0.451	0.2	0.452	0.0	0.497	9.8	0.536	18.6	0.205	54.6
20	0.722	0.707	2.1	0.461	36.1	0.974	34.9	0.704	2.5	0.612	15.2
21	1.323	1.254	5.2	0.831	37.2	1.956	47.8	1.288	2.7	0.449	66.1
22	0.598	0.608	1.7	0.429	28.3	0.772	29.2	0.634	6.1	0.517	13.6
23	0.169	0.158	6.3	0.080	52.6	0.212	25.6	0.124	26.3	0.138	18.5
24	0.221	0.215	2.6	0.106	52.2	0.337	52.4	0.169	23.4	0.134	39.5
25	1.450	1.561	7.7	1.518	4.7	1.519	4.8	1.583	9.2	0.725	50.0
26	0.176	0.171	2.8	0.127	27.9	0.251	42.7	0.191	8.6	0.080	54.9
27	0.181	0.172	4.7	0.127	29.8	0.251	39.0	0.191	5.8	0.080	56.0
28	0.190	0.174	8.6	0.127	33.3	0.251	32.0	0.191	0.5	0.080	58.2
29	0.197	0.175	11.0	0.127	35.4	0.251	27.8	0.191	2.7	0.080	59.6
30	0.207	0.175	15.4	0.127	38.7	0.251	21.3	0.191	7.6	0.080	61.6
			5.1		29.2		39.9		12.9		48.9

Table 2. Experimental values of *ETC* and other data for suspension systems. Values of thermal conductivities are in W m^{-1} K^{-1}: sphericity $\psi = 1$

SN	Sample Solid phase/fluid phase	Th. cond. solid (λ_2)	Th. cond. fluid (λ_1)	*ETC* exp (λ_e)	Vol. frac. cont. phase (ϕ)	Ref.
1	Graphite/water	160.028	0.664	0.830	0.950	[24]
2	Graphite/water	160.028	0.664	1.129	0.890	[24]
3	Graphite/water	160.028	0.664	1.435	0.830	[24]
4	Graphite/water	160.028	0.664	1.917	0.760	[24]
5	Selenium/polypropylene glycol	5.192	0.140	0.180	0.900	[25]
6	Selenium/polypropylene glycol	5.192	0.140	0.219	0.800	[25]
7	Selenium/polypropylene glycol	5.192	0.140	0.315	0.700	[25]
8	Selenium/polypropylene glycol	5.192	0.140	0.422	0.600	[25]
9	Aluminium/water	204.237	0.657	0.761	0.945	[26]
10	Aluminium/water	204.237	0.657	0.969	0.885	[26]
11	Aluminium/water	204.237	0.657	1.402	0.825	[26]
12	Aluminium/water	204.237	0.657	1.806	0.790	[26]
13	Graphite/water	160.941	0.562	1.185	0.839	[26]
14	Zinc sulphate/lard	0.613	0.197	0.309	0.625	[27]
15	Zinc sulphate/lard	0.613	0.197	0.351	0.445	[27]
16	Marble/vaselene	2.977	0.185	0.747	0.400	[27]

Calculation-table. Values of *ETC* are in W m^{-1} K^{-1}

SN	*ETC* exp (λ_e)	Proposed model equation (8) *ETC*	%Err	Brailsford model equation (A1) *ETC*	%Err	Litchnecker model equation (A2) *ETC*	%Err	Hadley's model equation (A3) *ETC*	%Err	Pande's model equation (A4) *ETC*	%Err
1	0.830	0.774	6.8	0.767	7.5	0.874	5.2	0.783	5.7	0.197	76.3
2	1.129	1.037	8.1	0.907	19.7	1.214	7.5	0.942	16.5	0.792	29.9
3	1.435	1.390	3.1	1.066	25.7	1.687	17.6	1.125	21.6	1.418	1.2
4	1.917	1.884	1.7	1.283	33.1	2.477	29.2	1.374	28.3	2.200	14.8
5	0.180	0.167	7.0	0.183	1.5	0.201	11.6	0.188	4.7	0.170	5.5
6	0.219	0.212	3.4	0.235	7.4	0.288	31.7	0.248	13.2	0.236	7.7
7	0.315	0.272	13.5	0.301	4.5	0.414	31.4	0.322	2.2	0.309	1.8
8	0.422	0.354	16.2	0.386	8.5	0.594	40.8	0.418	1.0	0.397	5.9
9	0.761	0.804	5.6	0.771	1.3	0.901	18.4	0.787	3.5	0.138	81.9
10	0.969	1.123	15.9	0.910	6.1	1.271	31.2	0.948	2.2	0.823	15.1
11	1.402	1.536	9.6	1.070	23.7	1.794	27.9	1.131	19.3	1.544	10.1
12	1.806	1.809	0.2	1.175	35.0	2.193	21.4	1.251	30.7	1.986	9.9
13	1.185	1.193	0.6	0.882	25.6	1.397	17.9	0.929	21.6	1.155	2.5
14	0.309	0.280	9.5	0.305	1.2	0.302	2.4	0.310	0.3	0.264	14.7
15	0.351	0.344	1.9	0.373	6.2	0.370	5.4	0.378	7.8	0.280	20.4
16	0.747	0.619	17.1	0.741	0.8	0.980	31.2	0.794	6.3	0.376	49.7
			7.5		13.0		20.7		11.6		21.7

Table 3. Experimental values of *ETC* and other data for emulsion systems. Values of thermal conductivities are in W m^{-1} K^{-1}: sphericity $\psi = 1$

SN	Sample Dispersed phase/cont. phase	Th. cond. dispersed phase (λ_2)	Th. cond. cont. phase (λ_1)	*ETC* exp (λ_e)	Vol. frac. cont. phase (ϕ)	Ref.
1	Water/petroleum solvent	0.604	0.182	0.266	0.8	[28]
2	Water/petroleum solvent	0.616	0.161	0.220	0.8	[28]
3	Water/petroleum solvent	0.609	0.173	0.311	0.6	[28]
4	Water/mineral oil	0.609	0.149	0.234	0.8	[28]
5	Water/mineral oil	0.611	0.149	0.292	0.6	[28]
6	Cellosize/flexol plasticizer	0.550	0.166	0.190	0.9	[29]
7	Cellosize/flexol plasticizer	0.574	0.190	0.209	0.9	[29]
8	Cellosize/flexol plasticizer	0.550	0.166	0.235	0.7	[29]
9	Cellosize/flexol plasticizer	0.574	0.190	0.256	0.7	[29]
10	Cellosize/polypropyline glycol	0.550	0.150	0.182	0.9	[29]
11	Cellosize/polypropyline glycol	0.574	0.154	0.180	0.9	[29]
12	Cellosize/polypropyline glycol	0.550	0.150	0.234	0.7	[29]

Calculation-table. Values of *ETC* are in W m^{-1} K^{-1}

SN	*ETC* exp (λ_e)	Proposed model equation (8) *ETC*	%Err	Brailsford model equation (A1) *ETC*	%Err	Litchnecker model equation (A2) *ETC*	%Err	Hadley's model equation (A3) *ETC*	%Err	Pande's model equation (A4) *ETC*	%Err
1	0.266	0.239	10.3	0.234	12.0	0.231	13.0	0.240	9.9	0.214	19.5
2	0.220	0.218	0.8	0.213	3.2	0.211	4.3	0.219	0.4	0.190	13.8
3	0.311	0.303	2.5	0.289	7.1	0.286	8.0	0.300	3.7	0.233	25.1
4	0.234	0.205	12.3	0.199	14.8	0.197	15.6	0.206	12.0	0.175	25.1
5	0.292	0.280	4.2	0.263	9.9	0.262	10.3	0.275	5.8	0.200	31.4
6	0.190	0.190	0.1	0.189	0.7	0.187	1.5	0.191	0.7	0.184	3.2
7	0.209	0.215	3.1	0.214	2.3	0.212	1.5	0.216	3.5	0.210	0.7
8	0.235	0.248	5.7	0.241	2.5	0.238	1.2	0.248	5.5	0.208	11.5
9	0.256	0.275	7.5	0.268	4.8	0.265	3.4	0.275	7.5	0.237	7.2
10	0.182	0.174	4.4	0.172	5.4	0.171	6.1	0.175	3.8	0.166	8.6
11	0.180	0.179	0.6	0.177	1.6	0.176	2.4	0.180	0.0	0.171	5.1
12	0.234	0.232	0.7	0.224	4.3	0.221	5.3	0.232	0.9	0.188	19.6
			4.3		5.7		6.1		4.5		14.2

Table 4. Experimental values of *ETC* and other data for solid–solid mixtures. Values of thermal conductivities are in W m^{-1} K^{-1}: sphericity $\psi = 1$

SN	Sample Dispersed phase/matrix phase	Th. cond. dispersed phase (λ_2)	Th. cond. matrix phase (λ_1)	*ETC* exp (λ_e)	Vol. frac. matrix phase (ϕ)	Ref.
1	Lead powder/silicon rubber	34.616	0.384	0.462	0.95	[30]
2	Lead powder/sliicon rubber	34.616	0.384	0.663	0.84	[30]
3	Bismuth powder/silicon rubber	8.307	0.384	0.432	0.95	[30]
4	Bismuth powder/silicon rubber	8.307	0.384	0.590	0.84	[30]
5	Bismuth powder/silicon rubber	8.307	0.384	0.732	0.76	[30]
6	Forsterite/magnesia	4.605	28.261	24.702	0.87	[31]
7	Zinc oxide/methyl vinyl	23.027	0.174	0.377	0.85	[32]
8	Titanium oxide/methyl vinyl	7.955	0.174	0.461	0.75	[32]
9	Silica powder/dimethyl	1.675	0.176	0.230	0.90	[32]
10	Silica powder/methyl vinyl	1.675	0.174	0.251	0.85	[32]
11	Silica powder/methyl vinyl	1.675	0.174	0.293	0.75	[32]
12	Zinc oxide/synthetic rubber	23.027	0.167	0.429	0.79	[32]

Calculation-table. Values of *ETC* are in W m^{-1} K^{-1}

SN	*ETC* exp (λ_e)	Proposed model equation (8) *ETC*	%Err	Brailsford model equation (A1) *ETC*	%Err	Litchnecker model equation (A2) *ETC*	%Err	Hadley's model equation (A3) *ETC*	%Err	Pande's model equation (A4) *ETC*	%Err
1	0.462	0.455	1.5	0.443	4.2	0.481	4.1	0.461	0.2	0.295	36.2
2	0.663	0.776	17.0	0.595	10.3	0.789	19.0	0.662	0.2	0.647	2.4
3	0.432	0.426	1.5	0.437	1.1	0.448	3.7	0.451	4.4	0.409	5.4
4	0.590	0.568	3.8	0.571	3.2	0.628	6.4	0.622	5.4	0.534	9.5
5	0.732	0.709	3.2	0.689	5.8	0.803	9.7	0.770	5.3	0.633	13.5
6	24.702	24.165	2.2	24.140	2.3	22.242	10.0	24.445	1.0	23.450	5.1
7	0.377	0.378	0.2	0.264	30.1	0.362	4.0	0.293	22.4	0.292	22.7
8	0.461	0.406	11.9	0.334	27.6	0.452	1.9	0.381	17.3	0.357	22.5
9	0.230	0.209	9.0	0.218	5.1	0.220	4.1	0.227	1.1	0.206	10.6
10	0.251	0.228	9.1	0.239	4.6	0.244	2.6	0.253	1.0	0.217	13.6
11	0.293	0.279	4.8	0.293	0.0	0.306	4.6	0.317	8.3	0.246	16.2
12	0.429	0.493	14.9	0.297	30.9	0.470	9.5	0.338	21.2	0.399	7.1
			6.6		10.4		6.6		7.3		13.7

Table 5. Thermal conductivity and other data for porous metallic systems: Hadley [1] thermal conductivity brass ($\lambda_2 = 113$), thermal conductivity steel ($\lambda_2 = 12.4$), thermal conductivity air ($\lambda_1 = 0.0274$) (sphericity $\psi = 1$: values of *ETC* are in $\mathrm{W\,m^{-1}\,K^{-1}}$)

SN	Porosity (ϕ)	*ETC* exp (λ_e)	Proposed model equation (8) *ETC*	%Err	Brailsford model equation (A1) *ETC*	%Err	Litchnecker model equation (A2) *ETC*	%Err	Hadley's model equation (A3) *ETC*	%Err	Pande's model equation (A4) *ETC*	%Err
(Brass/air system)												
1	0.07	36.81	32.88	11	1.11	97	63.10	71	6.47	82	−0.07	100
2	0.12	32.70	19.22	41	0.63	98	41.61	27	3.68	89	0.03	100
3	0.18	20.51	12.43	39	0.40	98	25.25	23	2.32	89	0.15	99
4	0.23	13.98	9.40	33	0.30	98	16.66	19	1.72	88	0.26	98
(Steel/air system)												
5	0.14	4.55	4.89	7	0.51	89	5.27	16	2.47	46	0.05	99
6	0.20	3.38	3.46	2	0.35	90	3.65	8	1.74	49	0.09	97
7	0.25	2.55	2.72	7	0.27	90	2.69	5	1.36	47	0.12	95
8	0.31	2.00	2.11	5	0.21	90	1.86	7	1.04	48	0.16	92
				18		94		22		67		98

value of the resistivity formation factor has been found to depend upon sphericity. Resistivity formation factor for a system with angular particles will be greater. An increase in value of ψ will increase particle to particle conduction of heat.

3.3. *Ratio of thermal conductivities* (λ_2/λ_1)

As established through an electrical analog given by Tareev [9], the *ETC* is found to depend on the ratio (λ_2/λ_1). A higher conductivity ratio favours a larger fraction of *ETC* in a direction perpendicular to heat flow.

3.4. *Particle cross-section ratio* ($A_\perp/A_\parallel$)

The concept of parallel slabs for different phases is a way of describing the systems. If a system actually consisted of different slabs, the theory should also give the resultant thermal conductivity of the composite structure. To incorporate this view a factor 'particle cross-section ratio' is also introduced in the expression for θ. If $A_\perp$ and $A_\parallel$ are respective areas of the slab perpendicular and parallel to the direction of heat flow, then its value is given by ($A_\perp/A_\parallel$). For spherical particles it is equal to one. For a dispersed system where particles are oriented in all possible directions, its average value is also one. For accounting the *ETC* of anisotropic systems, this factor may have some functional form.

While selecting a proper function for θ the following points were also taken into consideration.

(i) From electrical analogy, when the lines of electric force pass from one dielectric medium to another then their directions are related by tangents of their respective angles at the interface.

(ii) When slabs are in the direction of heat flow or perpendicular to it, then the expression representing θ should reduce to zero or $\pi/2$ respectively.

(iii) For $\phi = 0$, *ETC* should be $\lambda_e = \lambda_2$ (thermal conductivity of the dispersed phase). For $\phi = 1$, heat transported by the second phase is zero, so we should have $\lambda_e = \lambda_1$ (thermal conductivity of the continuous phase).

(iv) When the ratio $\lambda_2/\lambda_1 \gg 1$, the parallel component of *ETC* should be higher.

(v) Also when $A_\perp \ll A_\parallel$, θ approaches zero and the expression should represent the parallel configuration of slabs. For $A_\perp \gg A_\parallel$, it should represent the perpendicular configuration.

On these grounds, to facilitate the interpolation of the *ETC* between the parallel and perpendicular equivalent conductivity limits, a tangent function was found most suitable for defining the angle.

Accommodating all the parameters mentioned above, the most appropriate expression for the angle 'θ' was found to be.

$$\tan\theta = \left[B_0 \frac{A_\perp}{A_\parallel} F\phi^2 \left\{\psi \frac{\lambda_2}{\lambda_1}\right\}^{1/2}\right]$$

or

$$\theta = \tan^{-1}\left[B_0 \frac{A_\perp}{A_\parallel} F\phi^2 \left\{\psi \frac{\lambda_2}{\lambda_1}\right\}^{1/2}\right] \qquad (10)$$

where B_0 is a constant. For granular systems B_0 has been found to be 1.15, while its values for suspensions, emulsions, solid–solid mixtures and porous metallic systems are 1.7, 0.65 and 1.05 and 0.4, respectively.

3.5. *Prediction of the ETC* (λ_e)

With the knowledge of the parameters λ_1, λ_2, ϕ, ψ, B_0, etc. discussed above, now it is possible to predict the value of the *ETC* for various systems.

(1) First, the angle of inclination of the slabs 'θ' is obtained by using equation (10). The value of F is determined by equation (9).

(2) The above value of θ is used for calculating the two components λ_{pl} and λ_{pr}. The formulae being used are,

$$\lambda_{pl} = [\phi\lambda_1 + (1-\phi)\lambda_2] \cos\theta$$

$$\lambda_{pr} = \left[\frac{\phi}{\lambda_1} + \frac{(1-\phi)}{\lambda_2}\right]^{-1} \sin\theta$$

and

$$\lambda_e = [\lambda_{pl}^2 + \lambda_{pr}^2]^{1/2}. \qquad (8)$$

Now, using equation (8), effective thermal conductivity (λ_e) can be evaluated.

3.6. *Experimental determination of the particle cross-section ratio* ($A_\perp/A_\parallel$)

For a system where the particles are distributed in a random manner, the average value of this factor will be unity, therefore it need not be determined. However, for a system composed of fibres or having asymmetry in particle distribution, this ratio can be determined by viewing the area of particles through a microscope. A cross-section of the system parallel to the direction of heat flow is viewed by the microscope and the particles lying within a circle of radius R are counted. Let this number be 'n'. Therefore, the average area of the particles in the direction of heat flow will be,

$$A_\parallel = \frac{\pi R^2(1 - p\phi^{2/3})}{n}.$$

The term $p\phi^{2/3}$ represents the projected area of pores. Here $\phi^{2/3}$ is two dimensional porosity and 'p' is a factor converting the three dimensional porosity to its equivalent in two dimensions. The value of 'p' was estimated to be 0.246. Details of the method are given by Verma *et al.* [10].

The radius of the circle is so chosen that a countable number of particles (nearly 50 or 60) lie within it. The circle may be in the focal plane of the microscope eye piece or it may be a ring placed on the section of the sample itself. Depending upon the symmetry of the particles, the enclosing boundary of other shapes may also be taken.

Similarly by viewing the cross-section of the system perpendicular to the direction of heat flow, $A_\perp$ can also be determined and the ratio can be worked out.

4. RESULTS AND DISCUSSION

The proposed model has been tested on granular systems, suspensions, emulsions, solid–solid mixtures and packed metallic systems. The experimental data for these systems have been taken from the literature. These data along with the calculations are given in Tables 1–5. Here the system consisted of randomly dispersed particles, therefore the average value of the ratio ($A_\perp/A_\parallel$) has been assumed to be one.

A theoretical value of *ETC* is determined in each case and it is compared with those given by the Brailsford and Major [11], Hadley [1], Pande [12] and Litchnecker [13] models. Various expressions used in the calculations have been given in the Appendix.

Hadley [1] in his model has used an empirical constant 'f', which is supposed to lie between 0 and 1. For metallic powders he has given its value to be 0.947. However, this value does not suit the systems used by us for comparison. Therefore, the constants were determined so that the percentage deviation is least for the various systems. We found these values to be 0.82, 0.71, 0.75 and 0.75 for granular systems, suspensions, emulsions, and solid–solid mixtures, respectively.

Percentage deviation from the experimental value for each model has been determined and an average percentage deviation has been found. For various models, it has been mentioned at the bottom of each calculation table. It is found that the percentage error is least for the proposed model.

Its highest value is found to be 18 for packed brass disc (Table 5). The term giving rise to this large error is the sphericity. Hadley has used highly angular brass particles for formation of the packed brass discs (brass/air system). These particles have a sphericity value far less than one, whereas we have assumed it to be equal to one. If the sphericity of the particles be determined and substituted in equation (10), we expect a better agreement with the experimental values. Still, the average percentage error for the proposed model is least in comparison to that given by other theoretical models.

The experimental method given by Wadell for determination of the sphericity is a tedious one, so its exact determination is not possible. Moreover, it is not possible to determine the sphericity in all cases. For granular systems experimentally determined values of the sphericity have been used, hence the average deviation is smaller (5.1%) in comparison to that for suspensions. It can be seen that in emulsions, where the dispersed droplets are nearly spherical, the average percentage deviation is only 4.3%. For other models, the error is far greater, the highest being in the case of Pande's model. Various theoretical models considered here are not suitable to predict the *ETC* of metallic systems, whereas the proposed model is quite satisfactory in this respect also.

Besides, the success of the proposed model in predicting the *ETC*, it also has the following merits in its favour.

(1) It is a single expression which can be applied to any type of system for estimation of the *ETC*.

(2) The model is capable of predicting *ETC* close to the experimental values even for mixtures of higher conductivity ratio and high porosities, whereas the other models show higher deviations.

(3) It has potential for further developments to predict *ETC* of consolidated systems and fiber reinforced composites.

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APPENDIX

Various expressions used in calculation:

(1) Brailsford and Major model (two phase systems),

$$\lambda_e = \frac{\left[\lambda_1\phi_1 + \lambda_2\phi_2 \frac{3\lambda_1}{2\lambda_1+\lambda_2}\right]}{\left[\phi_1 + \phi_2 \frac{3\lambda_1}{2\lambda_1+\lambda_2}\right]}. \tag{A1}$$

(2) Litchnecker model (two phase systems),

$$\lambda_e = (\lambda_1)^{\phi_1}(\lambda_2)^{\phi_2}. \tag{A2}$$

(3) Hadley model (two phase systems),

$$\lambda_e = \frac{\lambda_1\left[\phi_1 f + \frac{\lambda_2}{\lambda_1}(1-\phi_1 f)\right]}{\left[1-\phi_1(1-f) + \frac{\lambda_2}{\lambda_1}\phi_1(1-f)\right]}. \tag{A3}$$

(4) Pande's model (two phase systems),

$$\lambda_e = \lambda_{EC}\left[1 - 3.8486\frac{(\lambda_{EC}-\lambda_1)}{(2\lambda_{EC}+\lambda_1)}\xi^{2/3}\right] \tag{A4}$$

where,

$$\lambda_{EC}^2 = \lambda_1\lambda_2\left[1 - 2.8366\left(\frac{m-2}{2m+5}\right) + 3.4011\left(\frac{m-2}{2m+5}\right)^2\right]$$

$$m = \left(\frac{\lambda_2}{\lambda_1}\right) \quad \text{and} \quad \xi = |(0.5-\phi)|.$$

λ and ϕ are the thermal conductivities and volume fractions of the phases, respectively.

Subscripts 1 and 2 represent the continuous and dispersed phase, respectively.